The exercises will take place in room G40 in Mühlenpfordtstrasse 23.

This week we will do a bit of theoretical work testing what you have learned so far. Complete the assignments and hand in your solutions to these theoretical tasks (with drawings/formulas). Please use different colors in your drawings. Each group hands in one solution. Your group must present the completed assignments on each Friday, 9:45.

9.1 Sketch kd-tree from point set (30 Points)
In the following you have to sketch a kd-tree for a given (2d) point set. At first, have a look at [http://homes.ieu.edu.tr/hakcan/projects/kdtree/kdTree.html](http://homes.ieu.edu.tr/hakcan/projects/kdtree/kdTree.html) to get an overview on kd-tree building for point sets.

In this task we will use the same algorithm for kd-tree building, i.e.: The first axis is vertical. The splitting axis at each level coincides with the median of a given dataset. For a vertical split the left child contains the points left of or on the splitting axis, the right child contains the points right of the splitting axis. For a horizontal split the left child contains the points below the or on the splitting axis, the right child contains the points above the splitting axis. A leaf node contains only one point. Now sketch the kd-trees for the following two point sets:

![Example kd-tree diagrams]

Also provide the tree structure with nodes and links. Name the points \( p_1, p_2 \), the splitting axis \( l_1, l_2 \)...

9.2 Guess the Fourier images (20 Points)
The following frequency domain images (second row) have been created for spatial domain images (first row) using numpy’s `fft2` method. However, the frequency domain images have been mixed up. Use your knowledge about the transformation properties of edges to guess, which frequency domain image belongs to which spatial domain image.
9.3 Fourier Transformation (20 Points)
The transformation of a signal $f(x)$ to Fourier space is given by

$$F(\omega) = \int_{-\infty}^{+\infty} f(x)e^{-2\pi i \omega x} dx$$

Consider the box function

$$B_d(x) = \begin{cases} 
0 & \text{for } x \leq -d \\
1 & \text{for } -d < x < d \\
0 & \text{for } d \leq x
\end{cases}$$

and show that its Fourier transformation is a sinc type function. Compare the behavior of the functions as $x \to \pm \infty$.

9.4 Inverse Fourier Transform and Image Processing (10 Points)
A checkerboard image $I$ has been applied the Fourier transform to get $g(x, y) = F\{I\}$. Then in Fourier space a multiplication with a 2d function $f(x, y)$ has been performed, where $f(x, y)$ is defined as

a) $f(x, y) = 1$ for $\sqrt{x^2 + y^2} \leq r$ and $f(x, y) = 0$ otherwise

b) $f(x, y) = 0$ for $\sqrt{x^2 + y^2} \leq r$ and $f(x, y) = 1$ otherwise

Sketch the resulting images when applying the inverse Fourier transform to $f \cdot g$ for both definitions of $f(x, y)$. Describe briefly what type of filtering has been realized for both situations.

9.5 Sampling Theory (20 Points)
Let $f(x)$ be an infinite signal. Consider a regular sampling $f_S(x)$ of $f(x)$ with sample distance $T$, that fulfills the Nyquist property, thus the highest frequency of the signal is smaller than $\frac{1}{2T}$

a) Is the exact signal reconstruction of $f(x)$ possible? If so, why?

b) How has the reconstruction to be performed in image and Fourier space?