The exercises will take place in room G40 in Mühlenpfordtstrasse 23.

From now on we will do some theoretical work testing and consolidating what you have learned so far. Complete the assignments and hand in your solutions to these theoretical tasks (with drawings/formulas). Please use different colors in your drawings. Each group hands in one solution on Friday, 9:45. Be prepared to present your solution in front of the class.

8.1 Barycentric coordinates (24 points)

You have already worked with Barycentric coordinates a lot. We are going to have a closer look at them.

a) A plane can also be expressed with the help of an arbitrary point on the plane and two linear independent vectors that span the plane. Give the expression in terms of the vertices.

b) Show that, if one Barycentric coordinate remains constant, and the other two vary, the point moves parallel to the side opposite of the constant coordinate.

c) Consider the point \( \vec{p}' \in \mathbb{R}^3 \) between \( \vec{a} \) and \( \vec{c} \), defined by the condition that \( (\vec{p}' - \vec{p}) \) is parallel to \( (\vec{b} - \vec{a}) \). Show that \( \lambda_3 = \frac{\|\vec{p}' - \vec{a}\|}{\|\vec{c} - \vec{a}\|} \).

d) Show that the Barycentric coordinates can be defined by a quotient of signed areas, for example \( \lambda_3 = \frac{A_{abc}}{A_{abp}} \).

8.2 Radiometry (Points 10)

Consider a perfectly spherical, diffuse light bulb with radius 3cm and a radiant power of 60W. Compute the emitted radiance \( L_e(x, \omega_o) \) on the surface of the light bulb.

*Hint:* Diffuse means that the radiance on any surface point and in any possible direction equals the same value \( L \).
8.3 Phong Model (Points 20 + 10)

Consider the reflection equation

\[ L_o(x, \omega_o) = \int_{\Omega^+} f_r(\omega_i, x, \omega_o)L_i(x, \omega_i) \cos(\theta_i) d\omega_i. \]

a) The BRDF of the simple Phong model is given by

\[ f_r(\omega_i, x, \omega_o) := k_s(\omega_i \cdot \omega_o)^n, \]

where \(0 < k_s < 1\) and \(n > 0\) are constants. Compute the radiosity \(\int_{\Omega^+} L_o(x, \omega_o)d\omega_o\) for light coming only from the normal direction, i.e. \(L_i(x, \omega_i) = \delta_n(\omega_i)\) and \(\omega_i \cdot \omega_o = \cos(\theta_o)\). The delta distribution \(\delta_n\) evaluates functions in the normal direction \(n\) with \(\theta_i = 0\), i.e.

\[ \int_{\Omega^+} g(\omega_i)\delta_n(\omega_i)d\omega_i = g(n). \]

**Hint:** Show that \(L_o(x, \omega_i) = k_s \cos(\theta_o)^n\). The integral over the upper half sphere \(\Omega^+\) can then be reduced to

\[ \int_{\Omega^+} L_o(x, \omega_o)d\omega_o = \int_0^{2\pi} \int_0^{\pi/2} k_s \cos(\theta_o)^n \sin(\theta_o) d\theta_o d\phi_o \]

using the relationship \(d\omega_o = \sin(\theta_o) d\theta_o d\phi_o\) for the differential angles. In order to compute the integral, derive the function \(\theta \mapsto (\cos \theta)^n\) using the chain rule.

b) How would the law of energy conservation for the reflection process look like? Give an informal as well as a mathematical formulation. Does the Phong model satisfy the energy conservation law?

8.4 Content comprehension (8 x 2 Points)

These are simple yes or no questions. Note that false answers count **negative** towards your total score!

a) The Phong reflectance model has a diffuse and a specular term

b) The Phong reflectance model is energy conservant

c) Bling-Phong is preferred over Phong because it is more efficient in terms computation time

d) Bling-Phong is preferred over Phong because it looks more nicely

e) The BRDF depends from wavelengths

f) The BRDF depends from the viewing angle

g) The BRDF depends from the distance to the object

h) The diffuse BRDF is anisotropic

8.5 Geometry and Topology (20 Points)

Show that there are exactly 5 platonic solids.