Physikbasierte Modellierung und Simulation
Assignment 10

Present your solutions for this sheet in the exercise on Thursday, February 2, 2017.

In this exercise, you will study the motion of a rotating rigid body. This is not a programming task!

Solve it on a piece of paper.

The problem is as follows: A dancer doing a pirouette rotates about the vertical axis with an angular velocity of \( |\omega| = 1 \text{s}^{-1} \) (Fig. 1). Suddenly, she moves her arms in towards her body (Fig. 2). What is the new angular velocity \( \omega' \)?

10.1 Inertia tensor (20 points)

Fig. 1 shows a crude approximation of the dancer as a system of point masses. Compute the inertia tensor \( I \) for the dancer when she rotates about the origin (her feet). You can read the positions \((x_i, y_i, z_i)\) and masses \(m_i\) of the body parts from the sketch; the numbers inside the dots are the masses of the respective body part in kilograms.

10.2 Angular momentum (20 points)

From the inertia tensor \( I \) and the angular velocity \( \vec{\omega} \), compute the angular momentum \( \vec{L} = I \vec{\omega} \) of the dancer before she moves her arms.
10.3 Modified inertia tensor (20 points)
Fig. 2 shows the dancer after having moved her arms inwards. Compute the inertia tensor $I'$ after the movement.

10.4 Conservation of angular momentum (20 points)
After the position of the arms has changed, the inertia tensor $I$ has changed to $I'$, but the angular momentum $\vec{L}$ remains constant because no external torques apply. This means that the angular velocity $\vec{\omega}$ has to change to some new value $\vec{\omega}'$ such that $\vec{L} = I'\vec{\omega}'$ is fulfilled. Compute the new angular velocity $\vec{\omega}'$.

10.5 Analysis (20 points)
You may have noticed that the direction of the axis of rotation has changed. If you neglect the actual movement of the arms and only look at the start and end positions, how could this happen? How would the movement of the dancer look like over the next few revolutions? Do you think the angular velocity of the dancer would really become this large? What could be a possible error in our model?