Physikbasierte Modellierung und Simulation
Assignment 8

Present your solutions for this sheet in the exercise on Wednesday, January 8, 2014. In this exercise, you will solve a system of linear equations using the conjugate gradient method. This is not a programming task! Solve it on a piece of paper.

8.1 Solve a system of linear equations (80 points)

Solve the system \( A\vec{x} = \vec{b} \) (see below) for \( \vec{x} \), starting from the zero vector. You should be able to see the solution of this system by just looking at it, so you can check whether you are on the right track. You can stop iterating as soon as the residual falls below \( 10^{-9} \).

\[
A = \begin{pmatrix} 1 & 1 & 4 \\ 1 & 1 & 1 \\ 4 & 1 & 1 \end{pmatrix},
\quad \vec{b} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}^T
\]

(All results within numerical precision.)

8.2 Analysis (20 points)

Answer the following questions:

a) What is the maximal number of iterations a conjugate gradient algorithm may take for a 3-by-3 system?

b) What is the principal advantage of the conjugate gradient algorithm compared to, for example, Gauss elimination?

c) What conditions does the matrix \( A \) have to fulfill?

d) Imagine you want to solve a system that does not have these properties. How could you still make use of the conjugate gradient method to solve it?

http://graphics.tu-bs.de/teaching/lectures/ws1314/pbm/